Let $\frac{\rho}{h}\sin A\cos(\delta-A)+p/c=D$, $\frac{\rho}{h}\sin(\delta-A)=E$.

$\therefore rD-p=\pm E\sqrt{(h^2-r^2\sin^2 A)}$.

$(D^2+E^2\sin^2 A)r^2-2Drp=E^2h^2-p^2$.

$\therefore r=\frac{Dp}{D^2+E^2\sin^2 A}$, by differentiation.

$\therefore D^2h^2+E^2h^2\sin^2 A=p^2\sin^2 A$.

Substituting D and E and reducing $\rho^2c^2\sin^2 A+2\rho chp\sin A\cos(\delta-A)$ $=p^2c^2\sin^2 A-p^2h^2$.

$\therefore \rho^2+2\rho H\cos(\delta-A)=G$ is the equation sought, and represents a circle.

Also solved by F. P. Matz.

243. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

What is the equation to the curve on which lie the centers of the inscribed circles in the right-angled triangles of hypotenuse h?

I. Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

Take as axes the hypotenuse and its perpendicular bisector. Denoting the length of the hypotenuse by h, and one acute angle by α, the equations to the bisectors of the acute angles may be written

$$y=(x+\tfrac{1}{2}h)\tan\tfrac{1}{2}\alpha,$$
$$y=-(x-\tfrac{1}{2}h)\tan(\tfrac{1}{4}\pi-\tfrac{1}{2}\alpha),$$
$$=-(x-\tfrac{1}{2}h)\frac{1-\tan\frac{1}{2}\alpha}{1+\tan\frac{1}{2}\alpha}.$$

By eliminating α between these equations we get as the locus of their intersection, that is, of the center of the inscribed circle, $4(x^2+hy+y^2)=h^2$.

II. Solution by L. S. SHIVELY, Mt. Morris College, Mt. Morris, Ill.

It is evident that the angle subtended by the hypotenuse and with vertex at the center of the inscribed circle equals 135°. Since it is constant and equal to 135°, its vertex, and hence the center of the circle, lies upon that arc of a circle constructed upon h as a chord, to contain an angle of 135°. Construct such a circle and join O, its center, with A and B, the ends of h. Then $\angle AOB=90°$. Also $OA=\frac{1}{2}h\sqrt{2}$. Referred to O as origin of coördinates, the equation of the circle is $2x^2+2y^2=h^2$.

Also solved by J. Scheffer, E. L. Sherwood, G. B. M. Zerr, and Elmer Schuyler.

244. Proposed by O. W. ANTHONY, Head of the Mathematical Department, DeWitt Clinton High School, New York.

Upon the sides of a triangle as bases isosceles triangles with base angles of 30° are constructed. Show that the lines joining the vertices of these isosceles triangles form an equilateral triangle.